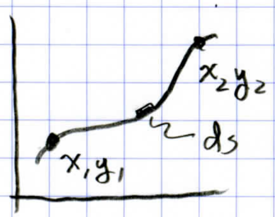
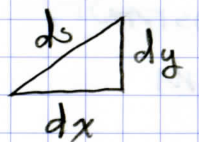
TM5 6.2

SHOW THAT THE SHORTEST DISTANCE BETWEEN TWO POINTS ON A PLANE IS A STRAIGHT LINE.





WRITE THE DISTANCE AS AN INTEGRAL

$$ds^2 = dx^2 + dy^2$$

$$ds = \sqrt{1 + \frac{dy^2}{dx^2}}\, dx$$

$$ds = \sqrt{1 + (y')^2}\, dx$$

$$\Rightarrow \quad s = \int_1^2 \underbrace{\sqrt{1 + (y')^2}}_{f(y, y'; x)}\, dx$$

APPLY EULER

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\frac{\partial f}{\partial y'} = 0$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\sqrt{1 + (y')^2} = 0$$

$$\frac{\partial f}{\partial y'} = \frac{\partial}{\partial y'}\sqrt{1 + (y')^2} = \frac{1}{\cancel{2}}\left(1 + (y')^2\right)^{-\frac{1}{2}}(\cancel{2}y')$$

THUS $\quad 0 - \dfrac{d}{dx}\left(\dfrac{y'}{\sqrt{1 + (y')^2}}\right) = 0$

AND $\quad \dfrac{y'}{\sqrt{1 + (y')^2}} = \text{CONSTANT} = C$

SOLVE FOR y'

$$(y')^2 = C^2\left[1 + (y')^2\right]$$

$\longrightarrow$

$$(y')^2 = c^2\left[1 + (y')^2\right]$$

$$(1 - c^2)(y')^2 = c^2$$

$$y' = \underbrace{\sqrt{\frac{c^2}{1-c^2}}}_{\text{JUST ANOTHER CONSTANT!}}$$

$\Rightarrow$ RE-NAME IT m

$$\frac{dy}{dx} = m$$

$$dy = m\,dx$$

$$\int dy = m \int dx$$

$$\Rightarrow \boxed{y = mx + \underbrace{b}_{\text{CONSTANT OF INTEGRATION}}}$$

EQUATION OF A LINE!

$b = y(x=0)$

b = INTERCEPT